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DIPARTIMENTO DI SCIENZE E TECNOLOGIE AEROSPAZIALI

Course of Introduction to Space Mission Analysis

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Analysis of different transfer strategies between two orbits

Group B.20

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1 | Introduction

This report has the purpose of defining and then analysing four different strategies to complete an orbital transfer between two assigned points on two completely different orbits. After characterising the orbits and the possible trajectories, at the end of the report various considerations will be made to find out which of them is optimal, first taking into account just the time to complete the transfer, then just the fuel that needs to be loaded on the spacecraft (in terms of absolute impulse) and then both requirements, to try and find an acceptable trade-off.

To achieve the scope of this report, different algorithms to calculate the required parameters were implemented in the MATLAB® environment [1], these algorithms will be attached to the report in order to allow for the verification of the numerical results contained within this report. After plotting the initial and final orbits in order to see their shape and position relative to each other, three transfer strategies will be defined, together with the standard strategy provided by the assignment.

The figure below (1.1) shows the reader the situation that will be taken in consideration in the following pages of this report, with the initial orbit shown in white and the final orbit shown in a light blue color. In addition to the orbits, the initial and final points required by the assignment and apsidal points are shown.

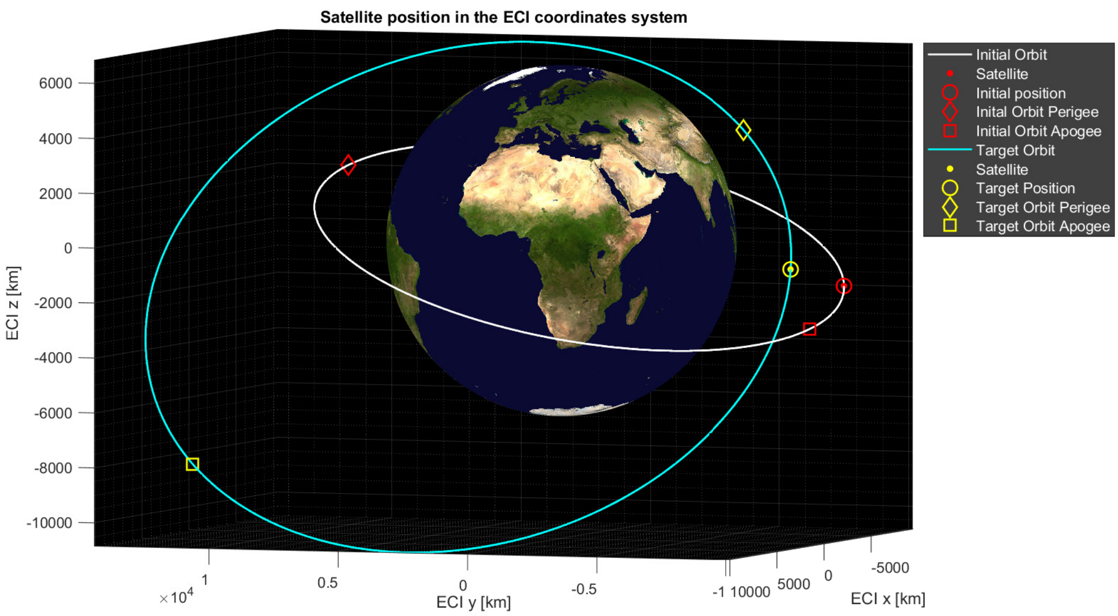


Figure 1.1: 3D Representation of the initial and final orbits

[1]To ensure compatibility and to avoid the insurgence of problems when running the algorithms it is recommended to use the latest MATLAB® build with the following add-ons installed: Symbolic Math Toolbox, Optimization Toolbox, Image Processing Toolbox, Image Acquisition Toolbox, Curve Fitting Toolbox

2 | Initial orbit characterisation

2.1. Orbital parameters

For the initial orbit, the Cartesian elements that define a position along the orbit were provided as data:

Initial Cartesian Orbital Parameters					
X (km)	Y (km)	Z (km)	v_x $(\frac{km}{s})$	v_y $(\frac{km}{s})$	v_z $(\frac{km}{s})$
−3719.30610	−9567.41250	−1666.47840	5.34000	−2.00800	−1.80800

Starting from the cartesian elements of the initial position, the Keplerian elements of the initial orbit were obtained using the *car2kep* function.

Initial Keplerian Orbital Parameters					
a (km)	e	i (deg)	Ω (deg)	ω (deg)	θ (deg)
9759.46565	0.07572	19.68167	41.76403	56.33149	152.07961

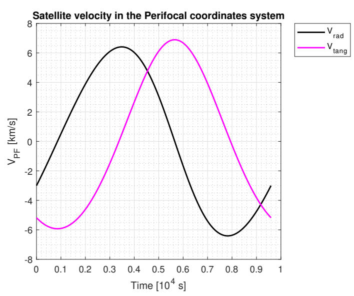
2.2. Discussion of the orbital parameters

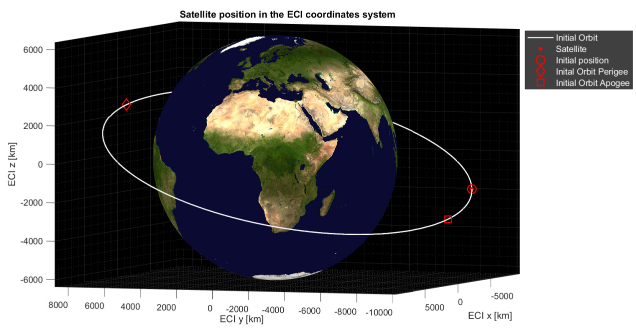
From the Keplerian orbital parameters the following information can be deduced:

- The semi-major axis (a) can be useful to classify the size of the orbit. After a quick calculation it's easily found that the altitude of both the perigee and apogee exceed 2000km (which is considered as the upper limit for LEO orbits). More precisely the altitude at the perigee is about 2600 km which can still be classified as in the LEO region, however the apogee altitude is more or less 4100 km which is closer to the start of the MEO region.
- Eccentricity (e) describes the elongation of the orbit compared to a circular one. Since for elliptical orbits e ranges from 0 (excluded) to 1 (excluded), the initial orbit is very close to being circular.

2.3. Graphical Representations

Note that the instant defined by $t = 0s$ is associated to the starting position along the orbit (θ) marked in the 3D representation by the hollow circle.





3 | Final orbit characterisation

3.1. Orbital parameters

For the target orbit, the data provided is the Keplerian elements that characterize the orbit:

Final Keplerian Orbital Parameters					
a (km)	e	i (deg)	Ω (deg)	ω (deg)	θ (deg)
12060.0	0.33870	49.41188	80.67245	140.03089	47.84197

Starting from the Keplerian elements of the target orbit, it's possible to go back to Cartesian elements that indicate the target position and velocity along the orbit by using the *kep2car* function.

Final Cartesian Orbital Parameters					
X (km)	Y (km)	Z (km)	v_x $(\frac{km}{s})$	v_y $(\frac{km}{s})$	v_z $(\frac{km}{s})$
−631.63647	−8628.70552	−904.86773	4.82430	−1.29145	−5.80084

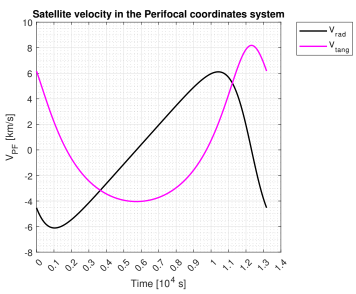
3.2. Discussion of the orbital parameters

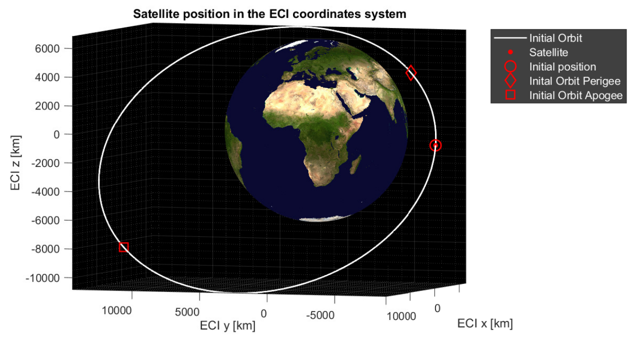
From the final Keplerian orbital parameters the following information can be deduced:

- The perigee and apogee altitudes of the final orbit are respectively around 1600 km and 9800 km. Thus we can infer that the orbit ranges from a LEO region to a MEO region, as the perigee is below the 2000 km upper limit for LEO and within the MEO region at its apoagee (the MEO region ranges between 5000 and 20000km). The semi-major axis (a) is greater than that of the initial orbit, however the perigee radius is smaller than the one from the initial orbit.
- Eccentricity (e): This final orbit has a more pronounced elliptical shape compared to the initial one. This can easily be noticed by looking at the difference between the perigee and apogee radius.

3.3. Graphical Representations

Note that the instant defined by $t = 0s$ is associated to the starting position along the orbit (θ) marked on the 3D representation by the hollow circle.





4 | Transfer trajectory definition and analysis

4.1. Methods to achieve the final position and velocity

In order to reach the target position it is necessary to modify the Keplerian elements of the initial orbit accordingly to get from the initial orbit to the final one. In particular it's necessary to modify the values of:

- a from 9759.46565 to 12060.0 km,
- e from 0.07572 to 0.33870,
- i from 19.68167 to 49.41188 degrees,
- Ω from 41.76403 to 80.67245 degrees,
- ω from 56.33149 to 140.03089 degrees.

To complete this transfer between two orbits so different in their Keplerian elements at least three maneuvers will be required, with many different combinations of maneuvers being possible. In this report four methods will be shown and analysed, including a standard method provided by the assignment.

Given that maneuvers are easier and less costly to achieve at apsidal points, once the spacecraft is on the target orbit, the final point will be reached without the need of another impulse, for this reason the proposed strategies will be considered completed once the target orbit is reached.

4.2. Possible transfer strategies

4.2.1. Standard Method: Change of plane → Change of Periapsis Argument at the best intersection point → Transfer from Perigee to Apogee

This method consists of a standard transfer strategy, provided by the assignment. It is composed of a change of the orbital plane as the first maneuver, followed by a change of the periapsis argument to align the obtained orbit with the final one.
Then, to reach the final orbit, a bi-tangent transfer maneuver starting from the perigee of the orbit obtained with the two previous maneuvers will be used to reach the final size and shape. This maneuver will reach the apogee of the target orbit, to then let the spacecraft freely reach the destination point.

The variations of all the Keplerian elements during the trajectory, the values of the impulses and the time required to complete this strategy can be seen in table A.2. A 3D representation of the initial/final orbits and of the spacecraft's trajectory can be seen in figure 4.1.

4.2.2. Method 1: Change of plane → Orbit Circularization → Transfer from Perigee to Apogee

This method is made out of three different maneuvers: as with all four strategies, the first one changes the orbital plane, to align with the final orbit's plane, then a circularization of the tilted orbit will be completed to avoid having to change the periapsis argument of the transfer orbit, because a circular orbit can have the perigee on whatever point is needed, thanks to the non-changing radius. After the second maneuver is completed, a transfer from the perigee to the apogee will be carried out, just like in the standard method.

The choice of circularizing the orbit instead of changing the periapsis argument is due to the low eccentricity of the initial orbit. By carrying out this maneuver, the ΔV may be lower and this strategy is used to check if that's true or not.

The variation of all the Keplerian elements during the trajectory, the values of the impulses and the time required to complete this strategy can be seen in table A.4. A 3D representation of the initial/final orbits and of the spacecraft's trajectory can be seen in figure 4.2.

4.2.3. Method 2: Change of plane → Orbit Circularization → Transfer from Apogee to Perigee

From the data obtained by analyzing the first proposed method, it's clear that a lot of time is being wasted to reach the maneuvering point on the circular orbit as doing the transfer from the perigee to the apogee implies that the spacecraft needs to travel over a big part of the circular orbit. Doing the transfer the other way around, so from apogee to pergigee, significantly decreases the time needed to reach the maneuvering point, thus reducing the total Δt.

The difference in ΔV due to the final maneuver happening at the perigee, where the speed is higher, should be balanced by the lower difference in speed between the circular orbit and the apogee of the bi-tangent transit orbit.

The variation of all the Keplerian elements during the trajectory, the values of the impulses and the time required to complete this strategy can be seen in table A.6. A 3D representation of the initial/final orbits and of the spacecraft's trajectory can be seen in figure 4.3.

4.2.4. Method 3: Change of plane → Orbit Circularization → Bi-elliptical Transfer

In this last method, just like in Method 1 and 2, a circularization will be carried out as it may be more convenient due to the low eccentricity of the initial orbit and because this way it's easier to choose the point at which to maneuver.
After that, instead of a probably more convenient bi-tangent transfer, a bi-elliptical maneuver with $r_b = 2000km$ will be carried out to check whether the intuition that this kind of maneuver may be less convenient than the other two methods, both time and ΔV wise, is actually confirmed by the data or not.

The variation of all the Keplerian elements during the trajectory, the values of the impulses and the time required to complete this strategy can be seen in table A.8. A 3D representation of the initial/final orbits and of the spacecraft's trajectory can be seen in figure 4.4.

4.3. Graphical Representations

4.3.1. Standard Method

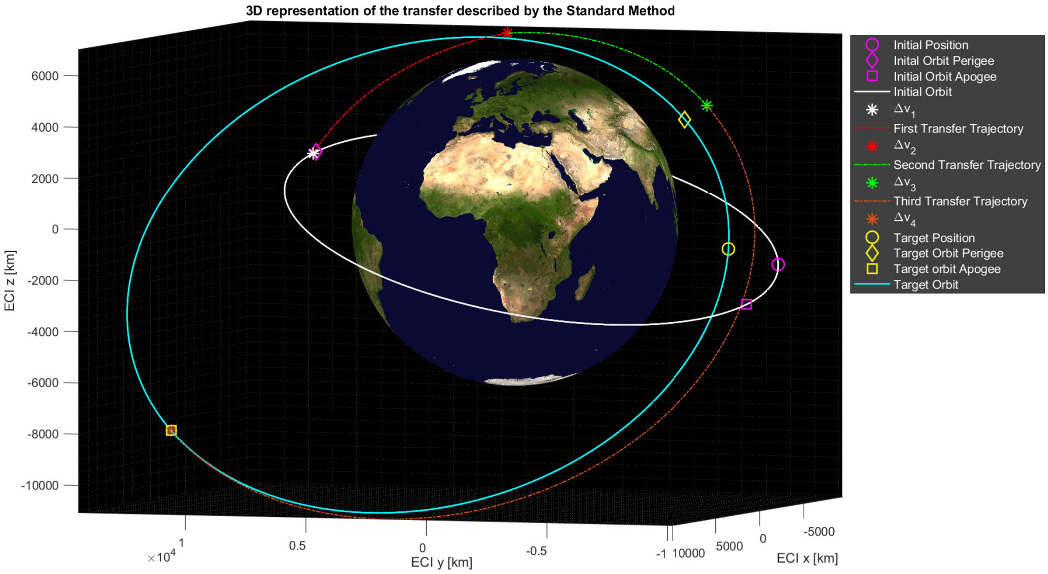


Figure 4.1: 3D representation of the trajectory described by the spacecraft in the Standard Method

4.3.2. Method 1

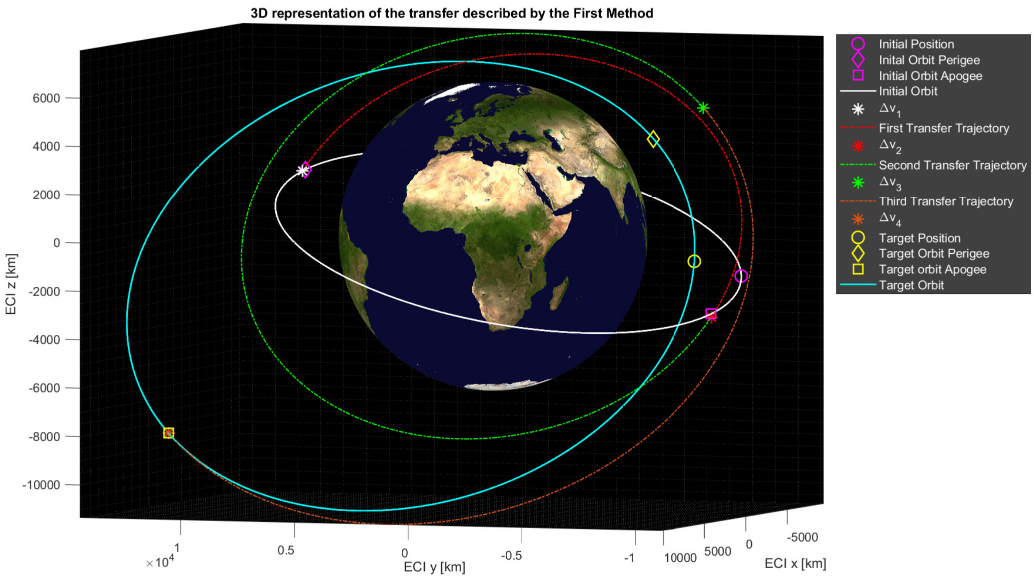


Figure 4.2: 3D representation of the trajectory described by the spacecraft in Method 1

4.3.3. Method 2

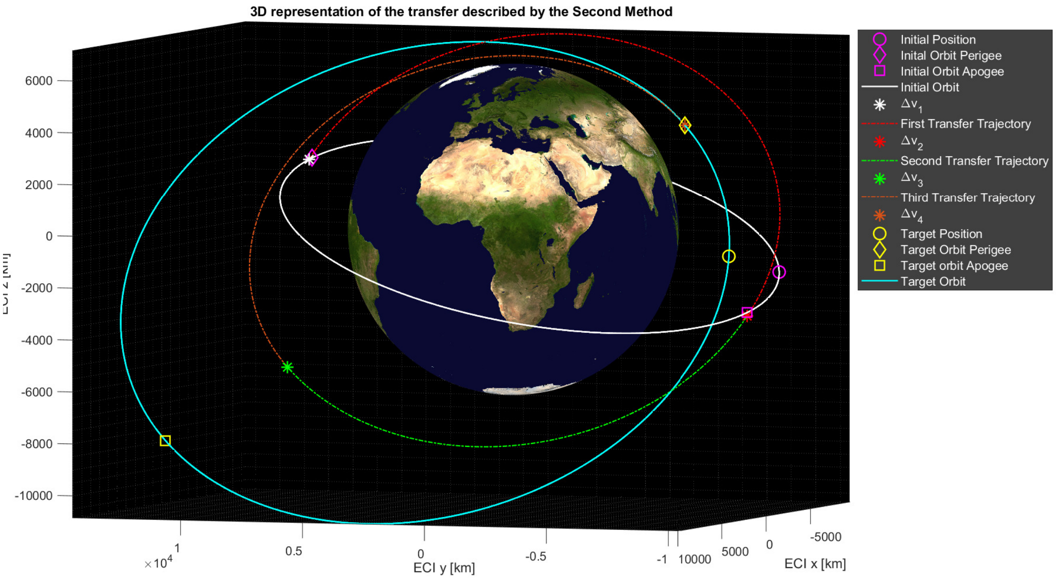


Figure 4.3: 3D representation of the trajectory described by the spacecraft in Method 2

4.3.4. Method 3

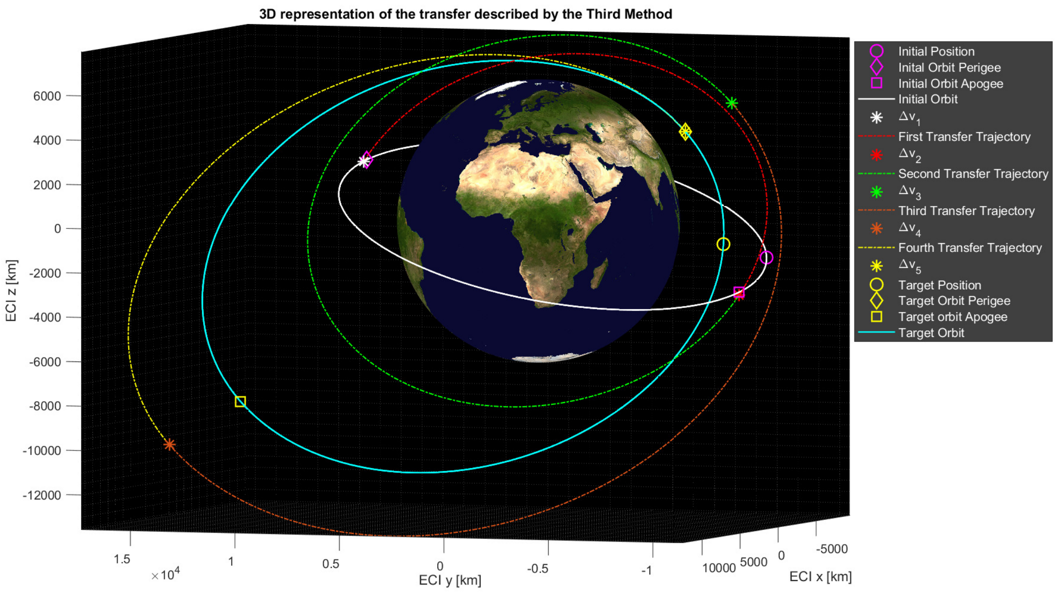


Figure 4.4: 3D representation of the trajectory described by the spacecraft in Method 3

5 | Conclusions

By analyzing each method in regards to time and ΔV requirements, we can classify the methods proposed in this report in order of convenience.
As is clearly deductible from the tables in the Appendix, method 2 is the best choice regarding the required Δt, at just 5 hours and 20 minutes, followed by the standard method clocking in at 6 hours and 20 minutes, a 19% increase over the best method.
These two methods can be very useful if the mission is time critical, so if it requires the spacecraft to reach the destination point as quickly as possible.
Method 1 and 3 are way worse than the other two methods, requiring respectively 9 hours and 33 minutes and 10 hours and 29 minutes, which is almost double the time required by method 2.

If the mission is weight, rather than time, restrained, focusing on choosing the strategy that requires the least fuel is obviously the way to go.
The best way to analyze which strategy requires the less fuel is to look at the total impulse required to complete all the maneuvers, as the value of the impulse is directly correlated to the amount of fuel required.
As is clearly visible from the tables in the Appendix, the strategy with the lower fuel requirement is, just barely, method 1 ($5.4616\frac{km}{s}$), with method 2 ($5.5485\frac{km}{s}$) coming close behind it with just a 1.59% increase and with method 3 and the standard one requiring respectively an increase of 9.33% and 7.32%.

The overall best, most balanced strategy out of the four proposed in this report is clearly method 2, which requires an hour less than the second-best method, balanced by just a $0.0869\frac{km}{s}$ increase in the required total impulse over the least impulse-heavy strategy.

The fact that method 3 is way worse than all other methods, is justified and explained in 5.1, which shows the ΔV required to complete a bi-tangent maneuver to the perigee, one to the apogee, and a bi-elliptical transfer related to the proportion between the perigee radius of the initial orbit over the one of the final orbit.
In the graph below, the vertical line shows the proportion between the perigee radii of the two orbits used in this report. As it is clearly deductible, both bi-tangent options are way more convenient a than bi-elliptical transfer, with the transfer to the perigee barely beating the transfer to the apogee.

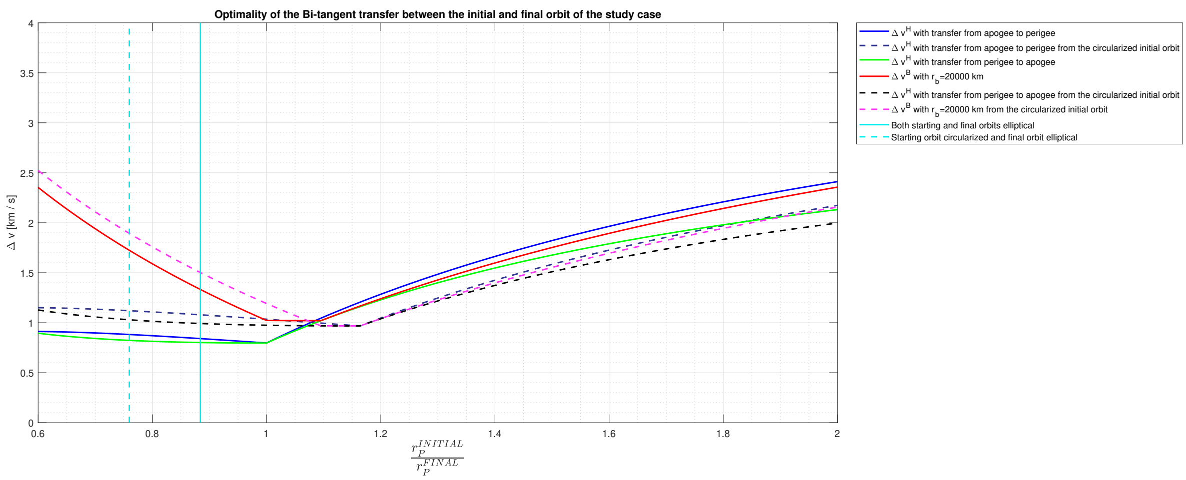


Figure 5.1: Optimality of different transfer maneuvers between the two orbits taken in consideration

A | Appendix

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv ($\frac{km}{s}$)
0	9759.46565	0.07572	19.68167	41.76403	56.33149	152.07961	–
5619.2	9759.46565	0.07572	19.68167	41.76403	56.33149	358.40229	4.2317
	9759.46565	0.07572	49.41188	80.67245	22.82664	358.40229	
7026.0	9759.46565	0.07572	49.41188	80.67245	22.82664	58.60212	0.8285
	9759.46565	0.07572	49.41188	80.67245	140.03089	301.39872	
8396.3	9759.46565	0.07572	49.41188	80.67245	140.03089	0.00000	0.6353
	12582.58282	0.28310	49.41188	80.67245	140.03089	0.00000	
15419.5	12582.58282	0.28310	49.41188	80.67245	140.03089	180.00000	(–)0.1663
	12060.0	0.33870	49.41188	80.67245	140.03089	180.00000	
22873.9	12060.0	0.33870	49.41188	80.67245	140.03089	47.84197	–
Total ΔT = 22873.9				Total $\Delta v = 5.8619 \frac{km}{s}$			

Table A.2: Standard Method

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv ($\frac{km}{s}$)
0	9759.46565	0.07572	19.68167	41.76403	56.33149	152.07961	–
5619.2	9759.46565	0.07572	19.68167	41.76403	56.33149	358.40229	4.2317
	9759.46565	0.07572	49.41188	80.67245	22.82664	358.40229	
10453.2	9759.46565	0.07572	49.41188	80.67245	22.82664	180.00000	0.2379
	10498.48766	0.00000	49.41188	80.67245	22.82664	180.00000	
19291.1	10498.48766	0.00000	49.41188	80.67245	140.03089	0.00000	0.6216
	13321.60482	0.21192	49.41188	80.67245	140.03089	0.00000	
26942.1	13321.60482	0.21192	49.41188	80.67245	140.03089	180.00000	(–)0.3704
	12060.0	0.33870	49.41188	80.67245	140.03089	180.00000	
34396.5	12060.0	0.33870	49.41188	80.67245	140.03089	47.84197	–
Total $\Delta T = 34396.5s$				Total $\Delta v = 5.4616 \frac{km}{s}$			

Table A.4: Method 1

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv ($\frac{km}{s}$)
0	9759.46565	0.07572	19.68167	41.76403	56.33149	152.07961	–
5619.2	9759.46565	0.07572	19.68167	41.76403	56.33149	358.40229	4.2317
	9759.46565	0.07572	49.41188	80.67245	22.82664	358.40229	
10453.2	9759.46565	0.07572	49.41188	80.67245	22.82664	180.00000	0.2379
	10498.48766	0.00000	49.41188	80.67245	22.82664	180.00000	
13938.5	10498.48766	0.00000	49.41188	80.67245	140.03089	180.00000	(–)0.4362
	9236.88282	0.13658	49.41188	80.67245	140.03089	180.00000	
18355.9	9236.88282	0.13658	49.41188	80.67245	140.03089	0.00000	0.6427
	12060.0	0.33870	49.41188	80.67245	140.03089	0.00000	
19220.1	12060.0	0.33870	49.41188	80.67245	140.03089	47.84197	–
Total ΔT = 19220.1s				Total Δv = 5.5485$\frac{km}{s}$			

Table A.6: Method 2

t (s)	a (km)	e (-)	i (deg)	Ω (deg)	ω (deg)	θ (deg)	Δv ($\frac{km}{s}$)
0	9759.46565	0.07572	19.68167	41.76403	56.33149	152.07961	–
5619.2	9759.46565	0.07572	19.68167	41.76403	56.33149	358.40229	4.2317
	9759.46565	0.07572	49.41188	80.67245	22.82664	358.40229	
10453.2	9759.46565	0.07572	49.41188	80.67245	22.82664	180.00000	0.2379
	10498.48766	0.00000	49.41188	80.67245	140.03089	180.00000	
19291.1	10498.48766	0.00000	49.41188	80.67245	140.03089	0.00000	0.8948
	15249.24382	0.31154	49.41188	80.67245	140.03089	0.00000	
28661.4	15249.24382	0.31154	49.41188	80.67245	140.03089	180.00000	(–)0.3332
	13987.63900	0.42983	49.41188	80.67245	140.03089	180.00000	
36893.3	13987.63900	0.42983	49.41188	80.67245	140.03089	0.00000	(–)0.2738
	12060.0	0.33870	49.41188	80.67245	140.03089	0.00000	
37757.5	12060.0	0.33870	49.41188	80.67245	140.03089	47.84197	–
Total ΔT = 37757.5s				Total Δv = 5.9714$\frac{km}{s}$			

Table A.8: Method 3